EXAM 2

1. A. GRADIENT

$$\nabla \varphi$$

B. DIVERGENCE

$$\nabla \cdot F$$

C. CURL $\nabla \times F$

D. $\displaystyle\int_C \varphi \, ds =$

$C \quad x = x(t) \quad y = y(t)$

$$\int_C \varphi \, ds = \int_{t_0}^{t_1} \varphi(x(t), y(t)) \sqrt{x'^2 + y'^2} \, dt$$

E. $F = \nabla \phi$

FOR SOME SCALAR FUNCTION

ϕ.

IF $u = x,\ v = y$

$\left\| \frac{\partial R}{\partial u} \times \frac{\partial R}{\partial v} \right\|$

$= \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2}$

2. $\iint_{\sigma} xy \, dS$

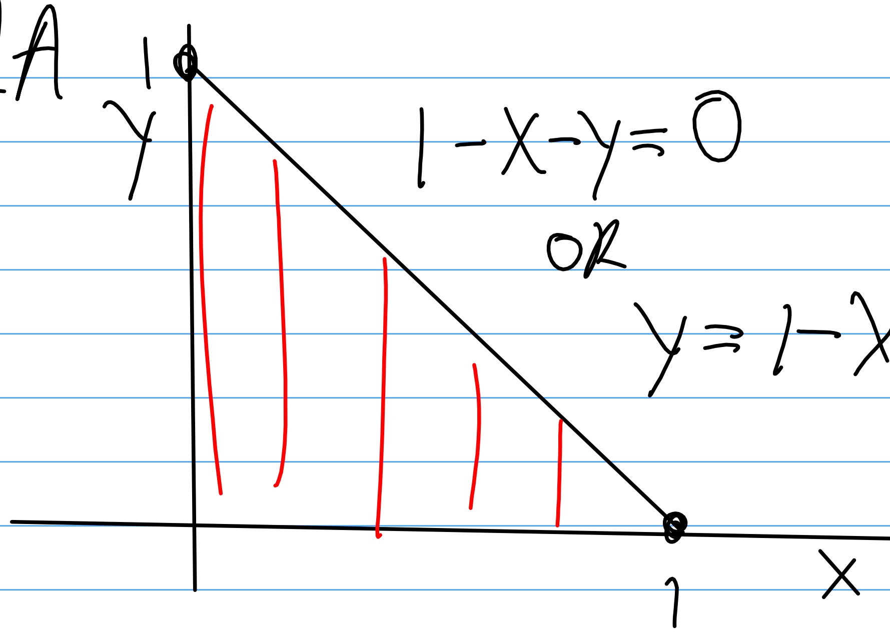
$S: \quad z = 1 - x - y$

$x, y, z \geq 0$

$$\iint_{\bar{\sigma}} xy\sqrt{1 + 1^2 + 1^2}\, dA$$

$\bar{\sigma}$ = REGION OF x, y VALUES

$$\sqrt{3}\iint xy\, dA$$



$$\sqrt{3}\int_0^1 \left\{ \int_0^{1-x} xy\, dy \right\} dx$$

$$\left. \frac{xy^2}{2} \right]_0^{1-x} = \frac{x(1-x)^2}{2}$$

$$= \frac{x-2x^2+x^3}{2}$$

$$\sqrt{3}\,\frac{1}{2}\int_0^1 x-2x^2+x^3\, dx = \frac{\sqrt{3}}{2}\left(\frac{x^2}{2}-\frac{2x^3}{3}+\frac{x^4}{4}\right)\Bigg]_0^1$$

$$\frac{\sqrt{3}}{2}\left(\frac{1}{2}-\frac{1}{3}+\frac{1}{4}\right)=\frac{\sqrt{3}}{2}\;\frac{5}{2}=\frac{5\sqrt{3}}{24}$$

FINAL ANSWER!

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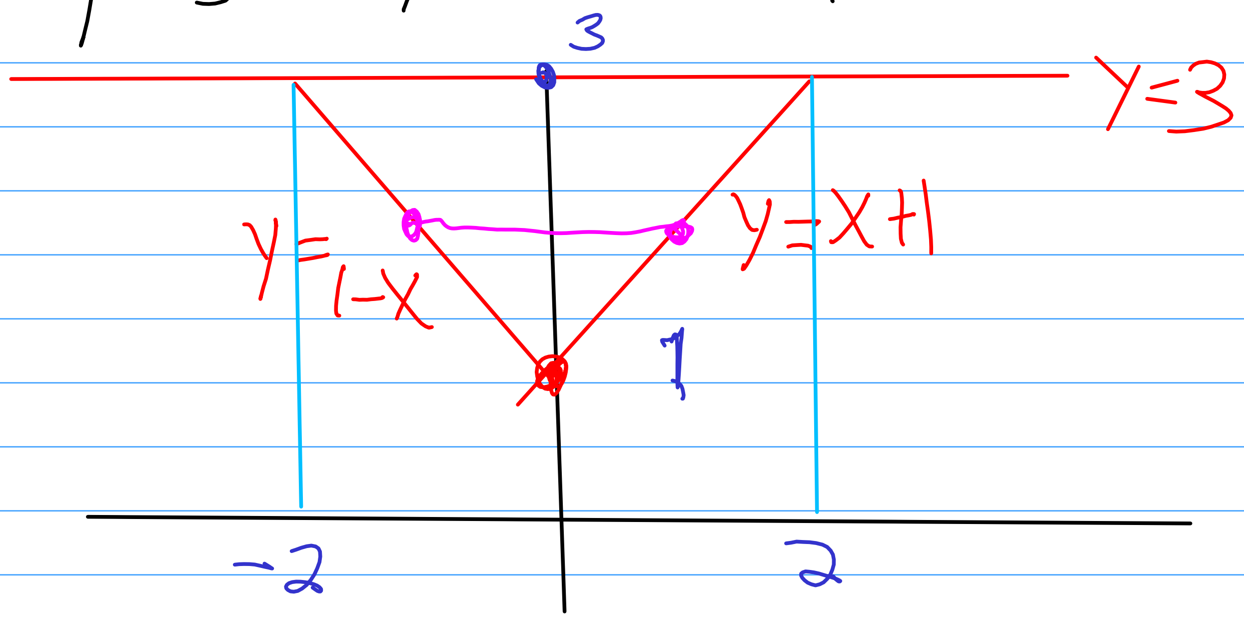
3. $$\iint_R (2x-y^2)\,dA$$

$y=3 \qquad y=-x+1 \qquad y=x+1$



$$\int_1^3 \left\{ \int_{1-y}^{y-1} (2x - y^2)\,dx \right\} dy$$

$$x^2 - xy^2 \Big]_{1-y}^{y-1} = \cancel{(y-1)^2} - (y-1)y^2 - \cancel{(1-y)^2} + (1-y)y^2$$

$$= -y^3 + y^2 + y^2 - y^3 = -2y^3 + 2y^2$$

$$\int_1^3 -2y^3 + 2y^2\,dy = -\frac{2y^4}{4} + \frac{2y^3}{3}\Big]_1^3$$

$$= -\frac{3^4}{2} + \frac{2\cdot 3^3}{3} + \frac{1}{2} - \frac{2}{3}$$

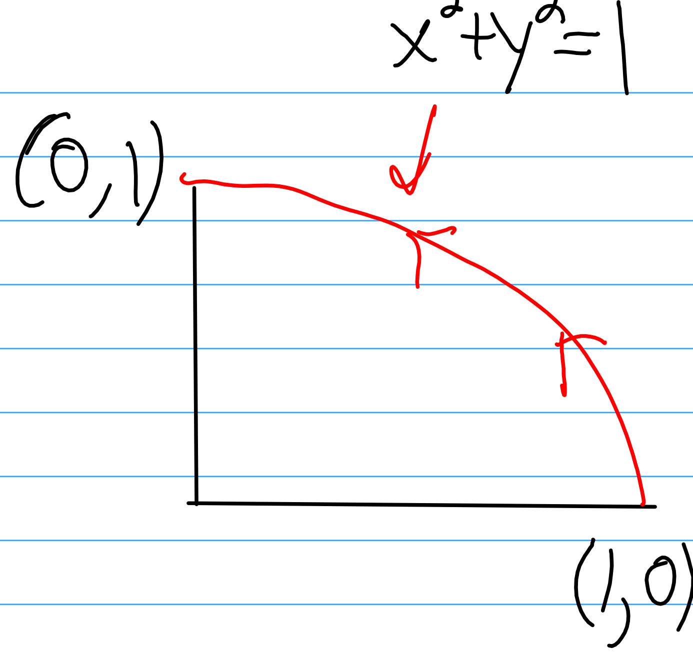
$$= -\frac{81}{2} + 18 + \frac{1}{2} - \frac{2}{3}$$

$$= -\frac{80}{2} + 18 - \frac{2}{3} = -40 + 18 - \frac{2}{3}$$

$$= -22\frac{2}{3}$$

FINAL ANSWER

4. $\int_C F \cdot dR$



$$F = \begin{bmatrix} x^2+y^2 \\ x \end{bmatrix}$$

$x = \cos\theta \qquad 0 \le \theta \le \frac{\pi}{2}$

$y = \sin\theta$

$$F \cdot \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 \\ \cos\theta \end{bmatrix} \cdot \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$$

$$= -\sin\theta + \cos^2\theta$$

$$\int_0^{\frac{\pi}{2}} -\sin\theta + \cos^2\theta \; d\theta$$

$$= \cos\theta + \frac{\theta + \sin\theta\cos\theta}{2}\Big]_0^{\pi/2}$$

$$\int \cos^2\theta\, d\theta = \sin\theta\cos\theta - \int \sin\theta(-\sin\theta)\, d\theta$$

$f' \quad g$

DERIVING $\int \cos^2\theta\, d\theta$

$$= \sin\theta\cos\theta + \int \sin^2\theta\, d\theta$$

$$\parallel$$

$$1 - \cos^2\theta\, d\theta$$

$$\int \cos^2\theta\, d\theta = \frac{\theta + \sin\theta\cos\theta}{2}$$

BACK TO OUR PROBLEM!

$$\int_C F \cdot dR = \cos\theta + \frac{\theta + \sin\theta\cos\theta}{2}\Big]_0^{\pi/2}$$

$$= 0 + \frac{\pi}{4} - 1$$

FINAL ANSWER!

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5. $$\oint_C F \cdot dR = \iint_{\text{DISK RADIUS 2}} \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) dA$$

$$\left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) = 2$$

$$F = \begin{bmatrix} x^2 - y \\ x \end{bmatrix}$$

$$= 2 \iint_{\text{DISK OF RADIUS 2}} dA = 8\pi$$

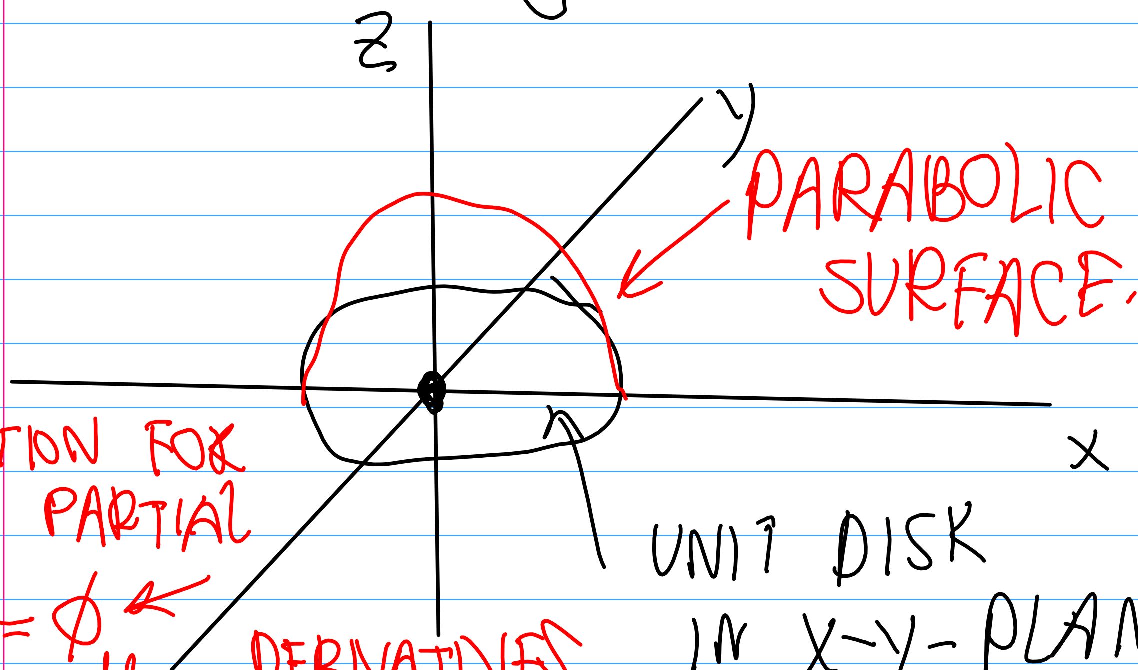
AREA OF THE DISK!

6. $$\iint_{\sigma} F \cdot N \, dS$$

$$F = \begin{bmatrix} x \\ y \\ 2z \end{bmatrix}$$

$z = 1 - x^2 - y^2$

$z \geq 0$



NOTATION FOR PARTIAL DERIVATIVES

$$\frac{\partial \phi}{\partial u} = \phi_u$$

$$\iint F \cdot N \, dS = \iint F \cdot \left(\frac{\partial R}{\partial u} \times \frac{\partial R}{\partial v} \right) du \, dv$$

IF $x = u$, $y = v$, $z = g(x, y)$

$$\iint F \cdot \begin{bmatrix} -g_x \\ -g_y \\ 1 \end{bmatrix} dx \, dy$$

$$N = \begin{bmatrix} -\frac{\partial z}{\partial x} \\ -\frac{\partial z}{\partial y} \\ 1 \end{bmatrix} = \begin{bmatrix} 2x \\ 2y \\ 1 \end{bmatrix}$$

$$F \cdot N = \begin{bmatrix} x \\ y \\ 2z \end{bmatrix} \cdot \begin{bmatrix} 2x \\ 2r \\ 1 \end{bmatrix} = 2x^2 + 2y^2 + 2z$$

ON THIS SURFACE!!!

$$z = 1 - x^2 - y^2$$

$$=$$

$$2x^2 + 2y^2 + 2(1 - x^2 - y^2) = 2$$

$$\iint_{x\text{-}y \text{ VALUES}} 2 \, dA = 2\pi$$

FINAL ANSWER!

7. USE THE DIVERGENCE THEOREM TO COMPUTE

$$\oiint_{\sigma} F \cdot N \, d\sigma = \iiint_{\text{UNIT BALL}} \nabla \cdot F \, dV$$

$$F = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

σ = SPHERE

$$x^2 + y^2 + z^2 = 1$$

$$\nabla \cdot f = \frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z} = 3$$

$$\nabla \cdot F = 3$$

$$\iiint_{\text{UNIT BALL}} 3 \, dV = 3 \times \text{VOLUME OF UNIT BALL} = 4\pi$$

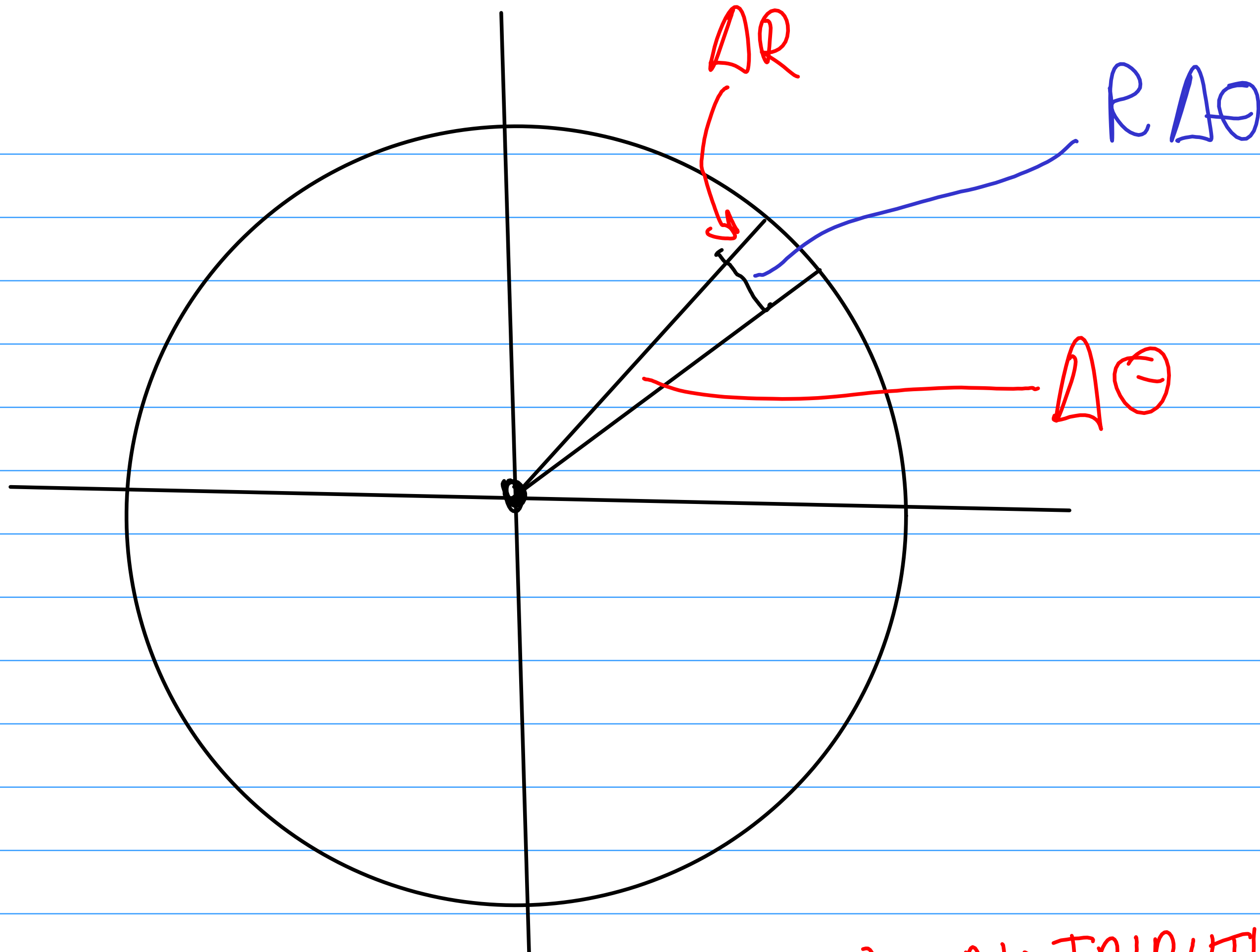
FINAL ANSWER 4π

$$\iint \varphi \, dA$$

CAN TRANSFORM INTO POLAR COORDINATES

$$dA = dx\,dy = r\,dr\,d\theta$$

LITTLE TWIST!



NORMAL DISTRIBUTION IN PROBABILITY THEORY!

$$\int_{-\infty}^{+\infty} e^{-x^2}\,dx = Z$$

$$\parallel$$

$$\int_{-\infty}^{+\infty} e^{-y^2}\,dy$$

$$Z^2 = \left(\int_{-\infty}^{+\infty} e^{-x^2}\,dx\right)\left(\int_{-\infty}^{+\infty} e^{-y^2}\,dy\right)$$

$$= \iint_{\text{WHOLE PLANE!}} e^{-x^2-y^2}\,dA$$

GO TO POLAR COORDINATES!

$$\iint_{\text{WHOLE PLANE}} e^{-R^2}\,R\,dR\,d\theta$$

HAH!

$$= \int_0^{2\pi} \left\{ \int_0^{\infty} e^{-R^2} R\, dR \right\} d\theta$$

$$\frac{1}{2}\int_0^{\infty} e^{-u}\, du = \frac{1}{2}$$

$u = R^2$

$du = 2R\,dR$

$$Z^2 = \int_0^{2\pi} \frac{1}{2}\, d\theta = \pi$$

$$\int_{-\infty}^{+\infty} e^{-x^2}\, dx = \sqrt{\pi}$$

$$\frac{1}{\sqrt{\pi}} e^{-x^2}$$

NORMAL DISTRIBUTION!

(INTEGRAL = 1)

PARTIAL DIFFERENTIAL
EQUATIONS

FOURIER SERIES

COMPLEX ANALYSIS.