

EXAM 1 — DISCRETE MATHEMATICS

Name: _____

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- (1) Prove by induction (10 pts each)
(a) $n^3 - n$ is divisible by 3

(b) $n^3 + 2n$ is divisible by 3

(2) The Lucas Sequence is defined by

$$L(1) = 1$$

$$L(2) = 3$$

$$L(n) = L(n-1) + L(n-2) \text{ for } n \geq 2$$

Prove that $L(n) = F(n+1) + F(n-1)$ for $n \geq 2$, where F is the Fibonacci sequence. (15 pts)

- (3) Describe each of these sets by listing its elements (5 pts each)
- (a) $\{x \mid x \in \mathbb{N} \text{ and } (\exists q(q \in \{2, 3\} \text{ and } x = 2q))\}$

(b) $\{x \mid x \in \mathbb{R} \text{ and } x^2 = 7\}$

(c) $\{x \mid x \in \mathbb{N} \text{ and } x^2 - 2x - 8 = 0\}$

- (4) For any finite set S , let $|S|$ denote the number of elements in S . Let $|A| = 3$ and $|B| = 4$. Find (5 pts each)
- (a) $|A \times B|$

(b) the maximum possible value for $|A \cap B|$

- (5) How many 3-digit numbers < 600 can be made using the digits 8, 6, 4, 2? (10 pts)

- (6) Concerning the set of bit-strings (i.e., each character is 0 or 1) of length 8:
- (a) How many such strings are there? (5 pts)

(b) How many begin and end with 0?(5 pts)

(c) How many begin or end with 0?(5 pts)

- (d) How many are palindromes (are the same on reflecting them)?(5 pts)

(e) How many contain exactly 7 1's? (10 pts)