

EXAM 2 — DISCRETE MATHEMATICS

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Show all work!

- (1) Given a 52 card standard deck and 5-card poker hands (a standard deck has 13 cards in each of 4 suits — clubs, diamonds, hearts, and spades):
 - (a) What is the probability of a pair? (10 pts)

(b) What is the probability of two pairs? (15 pts)

(2) A loaded die has the probability distribution

1	2	3	4	5	6
0.2	0.05	0.1	0.2	0.3	0.15

Let E_1 be the event that the number rolled is odd, let E_2 be the event that the number rolled is 3 or 6, and let E_3 be the event that the number rolled is ≥ 4 . (5 pts each)

(a) Find $P(E_1)$

(b) Find $P(E_2)$

(c) Find $P(E_3)$

(d) Find $P(E_1 \cap E_2)$

(e) Find $P(E_1 \cup E_2)$

(3) Use the binomial theorem to prove that:

$$C(n,0) + C(n,n-1)2 + C(n,n-2)2^2 + \cdots + C(n,n)2^n = 3^n$$

(10 pts)

(4) Consider the relation

$xpy \Leftrightarrow xy$ is a perfect square

(a) Is this reflexive? (5 pts)

(b) Is this symmetric? (5 pts)

(c) Is this transitive? (5 pts)

- (5) Find the transitive closures of the relations: (5 pts each)
- (a) $\rho = \{(1,2)\}$

$$(b) \rho = \{(1,2), (2,3), (1,3), (3,4)\}$$

- (6) Harry Potter wants to make an invisibility potion, with Hermione Granger and Ron Weasley helping (so 3 tasks can be performed at once). The steps involved are listed in the table:

Task	Number	Prerequisites	Duration
Acquire ingredients	1	none	3 days
Extract vorpall venom	2	1	2 days
Boil bats' eyes in baboon blood	3	1	5 days
Puree boiled bats' eyes in vorpall venom	4	2,3	2 days
Marinate pureed spiders in boiled baboon blood	5	2	4 days
Combine ingredients and freeze with mandrake root	6	4,5	2 days

- (a) How long does this potion take to produce? (5 pts)

(b) Which steps are critical? (10 pts)